1. Fig. 9 shows a sketch of the curve $y = x^3 - 3x^2 - 22x + 24$ and the line $y = 6x + 24$.

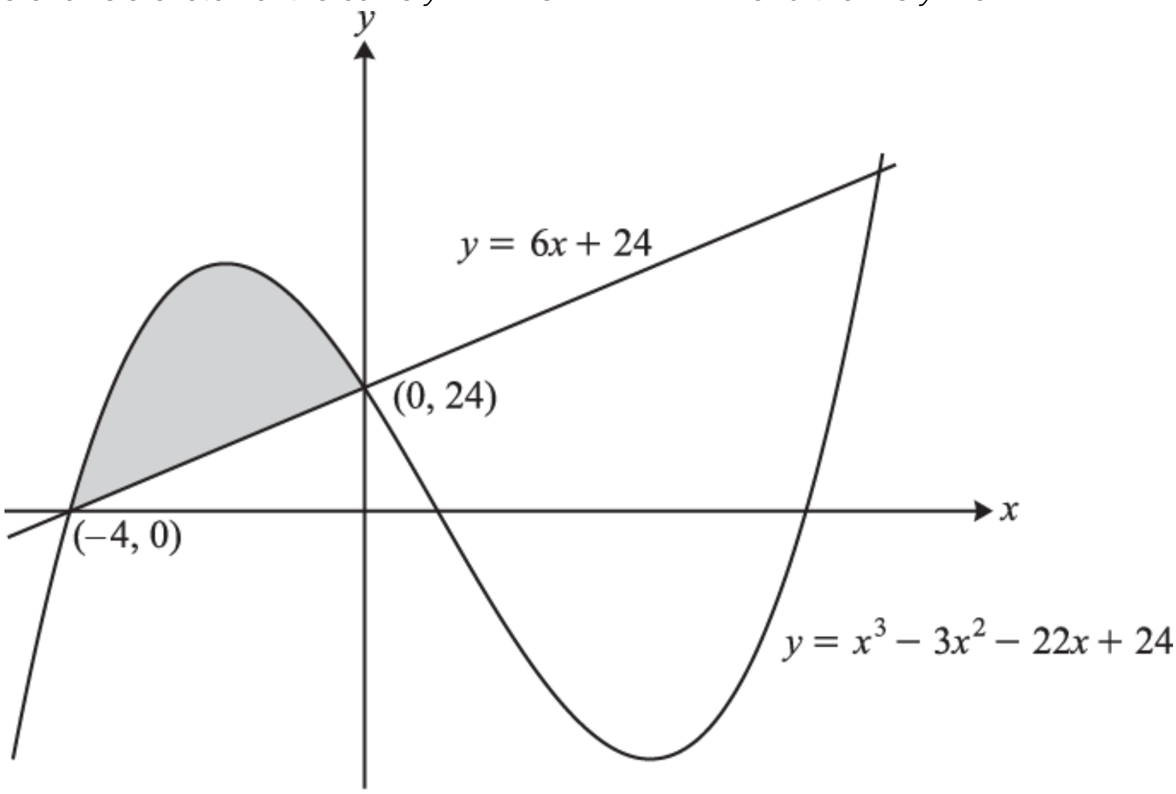

Fig. 9

i. Differentiate $y = x^3 - 3x^2 - 22x + 24$ and hence find the x-coordinates of the turning points of the curve. Give your answers to 2 decimal places.

[4]

ii. You are given that the line and the curve intersect when $x = 0$ and when $x = -4$. Find algebraically the x-coordinate of the other point of intersection.

[3]

iii. Use calculus to find the area of the region bounded by the curve and the line $y = 6x + 24$ for $-4 \leqslant x \leqslant 0$, shown shaded on Fig. 9.

[4]

2. Find $\int \left(x^2 + \frac{1}{x^2} \right) dx$. [3]

3. Show that the area of the region bounded by the curve $y = 3x^{-\frac{3}{2}}$, the lines $x = 1$, $x = 3$ and the x-axis is $6 - 2\sqrt{3}$. [5]

4. A curve passes through the point (4, 122) and its gradient is given by

$$\frac{dy}{dx} = 1 - \frac{4}{\sqrt{x}} + 6x^2.$$

Find the equation of the curve. [5]

5. **In this question you must show detailed reasoning.**

Find the total area of the shaded regions shown in Fig. 8, bounded by the line $x = -1$, the x-axis and the curve $y = x^3 (x - 3)$. [6]

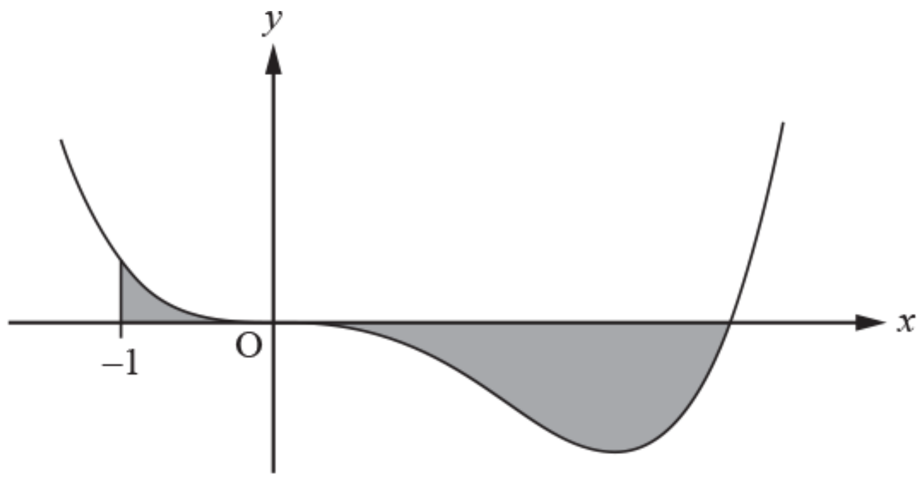

Fig. 8

6. (i) Find $\int_1^5 4x \, dx$ [3]

(ii) Find $\int 6x^{\frac{1}{2}} \, dx$ [2]

7. A curve passes through the point (2, 10) and has gradient $\frac{dy}{dx} = 12x^3 - 7$. Find the equation of the curve. [5]

8. Show that $\int_0^9 (3+4\sqrt{x})\,dx = 99$. [4]

9. Find $\int\left(4\sqrt{x} - \frac{6}{x^3}\right)dx$. [4]

10. **In this question you must show detailed reasoning.**

(a) Show that $x - 3$ is a factor of $4x^3 - 12x^2 - x + 3$. [1]

(b) Fig. 4 shows the curve $y = 4x^3 - 12x^2 - x + 3$. Find the coordinates of the points where it crosses the x– axis. [4]

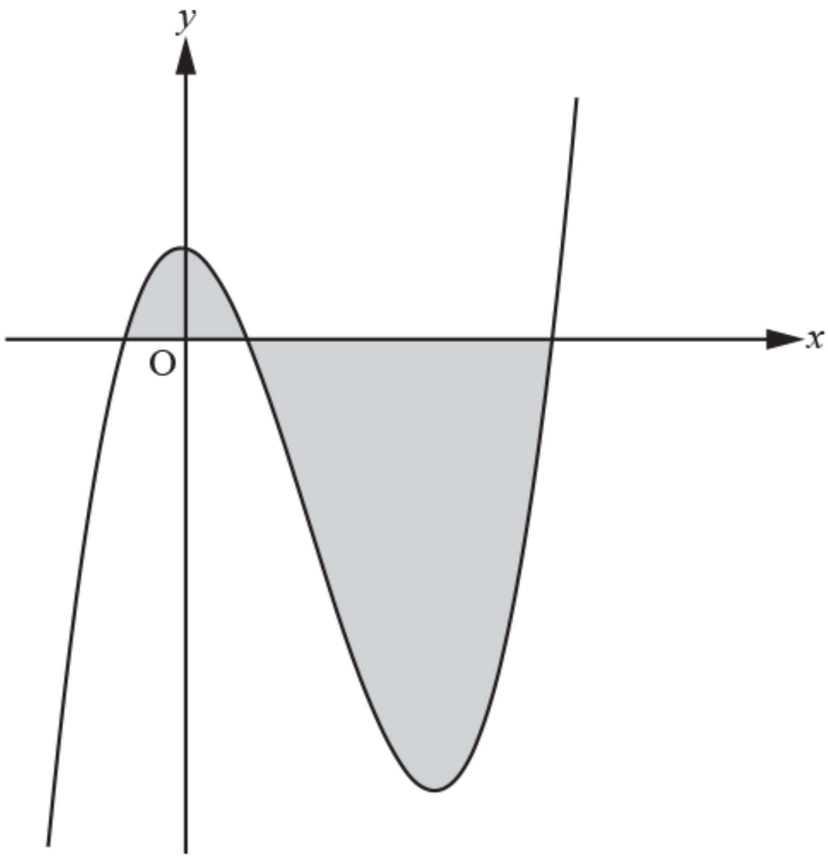

Fig. 4

(c) The two regions bounded by the curve $y = 4x^3 - 12x^2 - x + 3$ and the x–axis are shaded in Fig. 4.
Determine the total area of the shaded regions. [5]

11. **In this question you must show detailed reasoning.**

Fig. 8 shows the graph of a quadratic function. The graph crosses the axes at the points (–1, 0), (0, –4) and (2, 0).

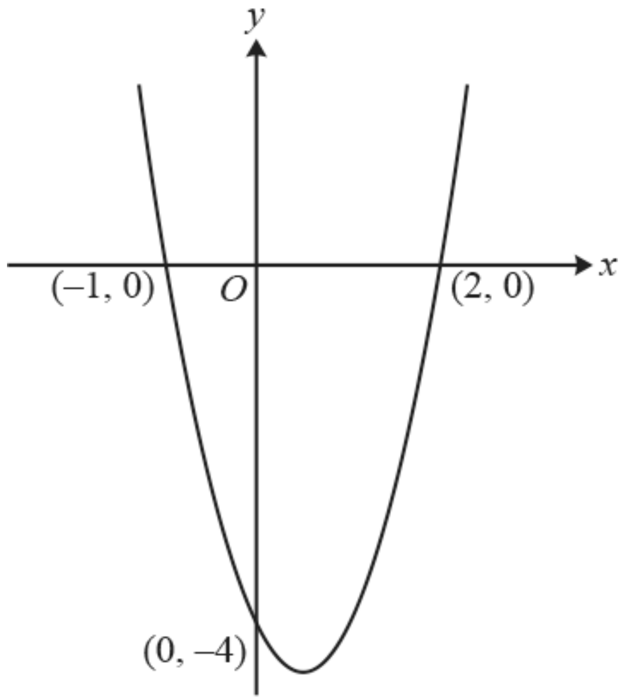

Fig. 8

Find the area of the finite region bounded by the curve and the x-axis. **[8]**

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$3x^2 - 6x - 22$	M1	condone one incorrect term, but must be three terms	condone "$y =$"
		i	their $y' = 0$ soi	M1	at least one term correct in their y'	may be implied by use of e.g. quadratic formula, completing square, attempt to factorise
		i	3.89	A1		
		i	−1.89	A1	if **A0A0**, **SC1** for $\frac{3 \pm 5\sqrt{3}}{3}$ or $1 \pm {}^5/_{\sqrt{3}}$ or better, or both decimal answers given to a different accuracy or from truncation __Examiner's Comments__ Nearly all candidates differentiated successfully and set their derivative to zero. Over 60% of candidates went on to score full marks, although a few candidates made an error (usually $2x^2$ but occasionally + 24 was retained). However, a significant minority attempted unsuccessfully to factorise the quadratic and then gave up and a surprising number were unable to use the quadratic formula correctly. Very few candidates appeared to check their answers. Some candidates lost an easy mark by leaving their answers in an exact form or by quoting a different precision. Occasionally, candidates found the second derivative and set this equal to zero. A significant minority wasted time either by finding the associated y-values or by determining the nature of the turning points, neither of which were required.	3.886751346 and −1.886751346
		ii	$x^3 - 3x^2 - 22x + 24 = 6x + 24$	M1	may be implied by $x^3 - 3x^2 - 28x$ [= 0]	
		ii	$x^3 - 3x^2 - 28x$ [= 0]	M1	may be implied by $x^2 - 3x - 28$ [= 0]	

					dependent on award of both M marks Examiner's Comments	
		ii	other point when $x = 7$ isw	A1	This was very well answered by most candidates. Well over 80% earned the first mark and most went on to score full marks. Occasionally, candidates slipped up when collecting like terms and a few made a sign error when factorising. The minority who failed to score either omitted the question altogether, or set $6x + 24$ equal to the derivative.	ignore other values of x
		iii	$\mathrm{F}[x] = \frac{x^4}{4} - \frac{3x^3}{3} - \frac{22x^2}{2} + 24x$	M1*	allow for three terms correct; condone + c	**alternative method** **M1** for $\int((x^3 - 3x^2 - 22x + 24) - (6x + 24))dx$ may be implied by 2nd **M1**
		iii	F[0] – F[–4]	M1dep	allow 0 – F[–4], condone – F[–4], but do not allow F[–4] only	**M1*** for $\mathrm{F}[x] = \frac{x^4}{4} - \frac{3x^3}{3} - \frac{28x^2}{2}$ condone one error in integration
		iii	area of triangle = 48	B1		**M1dep** for F[0] – F[–4]
		iii	area required = 96 from fully correct working	A1	**A0** for – 96, ignore units, Examiner's Comments This question was accessible to most candidates, although a significant minority scored zero. Many candidates found the area of the triangle using ½×base×height. Most of those who used a base of –4 realised that a negative area was impossible and so removed the minus sign. Some used integration and more often than not were successful – sometimes after 'losing' a minus sign. Most candidates also integrated successfully, but some made no further progress, as they ignored the upper limit and then 'airbrushed' the minus sign. A good proportion of those who did integrate successfully then made errors with the arithmetic. Some	no marks for 96 unsupported

					candidates earned two marks by c and integrating correctly, but a similar proportion ignored the upper limit or made arithmetical slips.	
			Total	11		
2			$\frac{1}{3}x^3$ $-\frac{1}{x}$ oe $+ c$	B1(AO1.1) B1(AO1.1) B1(AO1.1) [3]		
			Total	3		
3			$\int_1^3 3x^{-\frac{3}{2}}\,dx$ $\left[-6x^{-\frac{1}{2}}\right]_1^3$ $\frac{-6}{\sqrt{3}} - \frac{-6}{\sqrt{1}}$ $\frac{-6}{\sqrt{3}} + 6$	M1(AO1.1a) A1(AO1.1) A1(AO1.1) M1(AO1.1) E1(AO2.1)	Attempt to integrate (ignore missing limits) Correct integration Correct limits seen at some point Substitution of limits (condone one error) Correct intermediate step using surds which follows from the | Do not award any A- marks if M0 is given Given answer must be seen to score E1	

			$6-2\sqrt{3}$ AG	[5]	substitution of limits and is not identical to given answer and completion	
			Total	5		
4			$[y=]x-\dfrac{4x^{\frac{1}{2}}}{\frac{1}{2}}+\dfrac{6x^3}{3}$	M1(AO2.1)	Must be three terms	
			$[y=]x-8\sqrt{x}+2x^3+c$	A1(AO1.1)	$8\sqrt{x}$ or $8x^{½}$	at least two terms correct
				A1(AO1.1)	All correct including + *c*	
			Substitution of $y = 122$ and $x = 4$ in their $y=x-8\sqrt{x}+2x^3+c$	M1(AO1.1)		
			$y=x-8\sqrt{x}+2x^3+6$	A1(AO1.1) [5]		
			Total	5		

5			DR Consider $\int_{-1}^{0} x^3(x-3)\,dx$ and $\int_{0}^{3} x^3(x-3)\,dx$ $\int x^3(x-3)\,dx = \int (x^4 - 3x^3)\,dx$ $\frac{x^5}{5} - \frac{3x^4}{4} (+c)$ $\left[\frac{x^5}{5} - \frac{3x^4}{4}\right]_{-1}^{0} = 0 - \left(\frac{(-1)^5}{5} - \frac{3(-1)^4}{4}\right) = \frac{19}{20}$ $\left[\frac{x^5}{5} - \frac{3x^4}{4}\right]_{0}^{3} = \left(\frac{3^5}{5} - \frac{3\times 3^4}{4}\right) - 0 = -\frac{243}{20}$ $\frac{19}{20} + \frac{243}{20} = \frac{131}{10}$	M1(AO 3.1a) M1(AO 1.1a) A1(AO 1.1a) A1(AO 1.1b) A1(AO 1.1b) B1(AO 1.1b) [6]	Splitting the integral into positive and negative regions with correct limits Attempting to integrate expanded form Correct indefinite integral seen Must be seen to use limits Must be seen to use limits www; not dependent on previous marks	
			Total	6		

6	i	$2x^2$ oe F[5] – F[1] 48 cao	B1 M1 A1 [3]	ignore + c for the first two marks where F[x] = kx^2 — no marks for 48 unsupported **A0** for 48 + c **Examiner's Comments** Most candidates successfully integrated and went on to obtain the correct answer. A few spoiled this by leaving "+ c" in the final answer, and a small number either differentiated or simply evaluated the integrand.	
	ii	$kx^{\frac{1}{2}+1}$ seen $4x^{\frac{3}{2}}+c$ or $4\sqrt{x^3}+c$ or $4\left(\sqrt{x}\right)^3+c$ isw	M1 A1 [2]	**Examiner's Comments** Nearly all candidates achieved the method mark by integrating, but a surprising number omitted the constant of integration thereby losing an easy mark.	
		Total	5		

7			kx^4 $3x^4$ $-7x + c$ $10 = (\text{their } 3) \times 2^4 - 7 \times 2 + c$ oe $y = 3x^4 - 7x - 24$	M1 A1 B1 M1 A1 [5]	<table><tr><td>$k > 0$ may be seen later must follow from integration must be 3 terms on RHS including term in x^4, term in x and "c"; or $y = 3x^4 - 7x + c$ and $c = -24$ stated isw</td><td>must **not** follow from use of $y = mx + c$ must **not** follow from use of $y = mx + c$ must see "$y =$" or "f(x) = " at some point for A1</td></tr></table> **Examiner's Comments** The vast majority of candidates tackled this question successfully. A few slipped up with the arithmetic in finding c, and a small minority worked with $y = mx + c$ with $m = 12 \times 3 - 7$ and failed to score.
			Total	5	

8			$F[x] = 3x + \dfrac{4x^{\frac{3}{2}}}{\frac{3}{2}}$ oe F[9] – F[0] $27 + \dfrac{4\times2\times27}{3} = 99$	M1(AO2.1) A1(AO1.1) B1(AO1.1) B1(AO2.4) [4]	Attempt at integration; sight of (first term) kx or (second term) $kx^{\frac{3}{2}}$ — Allow $+c$ Dep M1; ft their F(x); Accept 27 + 72 =99 AG — Must see convincing arithmetic for award of final mark **Examiner's Comments** Most candidates were able to integrate the expression, though the square root did cause some problems. Quite a number of candidates seemed uncertain as to the point at which the integral sign with limits before the expression should be replaced with square brackets to show that the expression should now be evaluated. In order to gain full marks candidates were required to show that they had worked out F(9) and F(0) and had calculated F(9) – F(0). They were also required to show that that the value of 99 for F(9) came from 27 + 72. The question asked for the given result to be shown; writing $\left[3x + \frac{8}{3}x^{\frac{3}{2}}\right]_0^9 = 99 - 0 = 99$ does not show this sufficiently well to be given full marks.	

			Total	4	Integration as the Reverse of Differentiation and Area Under a Curve (Yr. 1)	
9			$kx^{\frac{3}{2}}$ kx^{-2} $\frac{8}{3}x^{\frac{3}{2}}$ or $+ 3x^{-2}$ seen $\frac{8}{3}x^{\frac{3}{2}} + 3x^{-2} + c$ | isw	M1 (AO 1.1) M1 (AO 1.1) A1 (AO 1.1) A1 (AO 1.1) [4]	**Examiner's Comments** Generally answered well, but the + c was often missed.	
			Total	4		
10		a	DR 108 − 108 − 3 + 3 = 0	B1 (AO 2.4) [1]	Sub. $x = 3$ and correct completion	
		b	DR $(x - 3)(4x^2 - 1)$ $(x - 3)(2x - 1)(2x + 1)$ $(3, 0), \left(\frac{1}{2}, 0\right), \left(-\frac{1}{2}, 0\right)$	M1 (AO 1.1a) M1 (AO 1.1) A1 (AO 1.1) A1 (AO 2.2a)	For at least 2 correct x–values	

				[5]	For all three correct points		
	c		DR $\int_{-\frac{1}{2}}^{\frac{1}{2}}(4x^3-12x^2-x+3)\,dx-\int_{\frac{1}{2}}^{3}(4x^3-12x^2-x+3)\,dx$ $\left[x^4-4x^3-\frac{1}{2}x^2+3x\right]_{-\frac{1}{2}}^{\frac{1}{2}}-\left[x^4-4x^3-\frac{1}{2}x^2+3x\right]_{\frac{1}{2}}^{3}$ $\left(\frac{15}{16}-\frac{(-17)}{16}\right)-\left(-22\frac{1}{2}-\frac{15}{16}\right)$ $25\frac{7}{16}$	M1 (AO 3.1a) M1 (AO 1.1) A1 (AO 1.1) M1 (AO 1.1) A1 (AO 2.1) [5]	May be awarded at any stage for adding *their* two absolute areas Integration: at least one term correct All terms correct Use of *their* two pairs of limits awrt 25.4	Ignore limits (or absence of limits) for these marks	
			Total	10			
11			EITHER Equation of the form $y = k(x+1)(x-2)$	M1 (AO1.1a) M1	DR Allow with $k = 1$ and without $y =$ Attempt to find	Ignore = 0 if seen	

			Answer	Marks	Guidance		
			$(0, -4)$ on curve so $k = 2$	(AO3.1a)	$k \neq 1$		
			OR	A1 (AO1.1b)	All correct		
			Equation of the form $y = ax^2 + bx + c$				
			$(0, -4)$ on curve $c = 4$				
			$(-1, 0)$ on the curve $0 = a - b - 4$	(M1)	Uses one point to form an equation	Allow for $c = -4$ seen	
			$(2, 0)$ on the curve $0 = 4a - 2b - 4$	(M1)	Uses both other points and attempts to solve simultaneous equations		
			Solving simultaneous equations $a = 2$, $b = -2$	(A1)	All correct		
			BOTH				
			$\text{Area} = \int_{-1}^{2}\left(2x^2 - 2x - 4\right)\mathrm{d}x$				
			$\left[\frac{2x^3}{3} - x^2 - 4x\right]_{-1}^{2}$	M1 (AO1.1a)			
			$\left(\frac{2\times 2^3}{3} - 2^2 - 4\times 2\right) - \left(\frac{2\times(-1)^3}{3} - (-1)^2 - 4\times(-1)\right)$	A1 (AO1.1b) M1 (AO1.1a)	Integration – allow without limits – condone one error FT their quadratic		

			$= -\frac{20}{3} - \frac{7}{3} = -9$	A1 (AO2.1)	Substitution of limits clearly seen	
			Area is 9 below the *x*-axis.	E1 (AO2.4) [8]	Complete argument leading to exact answer. Allow for 9 if there is an argument to explain the change of sign even if -9 not seen. Must give modulus and explain the change of sign. FT if their definite integral is negative.	"Area must be positive" is not sufficient explanation.

Examiner's Comments

There were many very good answers to this question but many lost the final mark as they did not explain why their area is given as positive when the definitive integral gives a negative value. Only a few candidates used their calculators to evaluate their definite integral but lost marks as this was a detailed reasoning question that required all the lines of working to be clear. Examiners needed to see the indefinite integral and the substitution of limits. Some candidates made their answers unnecessarily complicated by splitting the

					required area into two or more regions. Many candidates struggled to obtain the correct equation of the curve, either using $y = (x + 1)(x + 2)$ or $y = x^2 - 4$ but most of the rest of the marks in this question were obtained following through their equation if it was quadratic. Make sure you do not write −9 = 9 without explaining the change of sign. Candidates needed to comment that the area is below the x-axis. Do not abandon a long question if there is a problem with the first part. Use any vaguely sensible equation to demonstrate your ability to integrate and use limits – it is not enough to describe this process in words.	
			Total	8		